Contents

Preface to the Second Edition ix
Preface to the First Edition xi

1 Hyperbolic Partial Differential Equations 1
   1.1 Overview of Hyperbolic Partial Differential Equations 1
   1.2 Boundary Conditions 9
   1.3 Introduction to Finite Difference Schemes 16
   1.4 Convergence and Consistency 23
   1.5 Stability 28
   1.6 The Courant–Friedrichs–Lewy Condition 34

2 Analysis of Finite Difference Schemes 37
   2.1 Fourier Analysis 37
   2.2 Von Neumann Analysis 47
   2.3 Comments on Instability and Stability 58

3 Order of Accuracy of Finite Difference Schemes 61
   3.1 Order of Accuracy 61
   3.2 Stability of the Lax–Wendroff and Crank–Nicolson Schemes 76
   3.3 Difference Notation and the Difference Calculus 78
   3.4 Boundary Conditions for Finite Difference Schemes 85
   3.5 Solving Tridiagonal Systems 88

4 Stability for Multistep Schemes 95
   4.1 Stability for the Leapfrog Scheme 95
   4.2 Stability for General Multistep Schemes 103
   4.3 The Theory of Schur and von Neumann Polynomials 108
   4.4 The Algorithm for Schur and von Neumann Polynomials 117
5 Dissipation and Dispersion
   5.1 Dissipation 121
   5.2 Dispersion 125
   5.3 Group Velocity and the Propagation of Wave Packets 130

6 Parabolic Partial Differential Equations 137
   6.1 Overview of Parabolic Partial Differential Equations 137
   6.2 Parabolic Systems and Boundary Conditions 143
   6.3 Finite Difference Schemes for Parabolic Equations 145
   6.4 The Convection-Diffusion Equation 157
   6.5 Variable Coefficients 163

7 Systems of Partial Differential Equations in Higher Dimensions 165
   7.1 Stability of Finite Difference Schemes for Systems of Equations 165
   7.2 Finite Difference Schemes in Two and Three Dimensions 168
   7.3 The Alternating Direction Implicit Method 172

8 Second-Order Equations 187
   8.1 Second-Order Time-Dependent Equations 187
   8.2 Finite Difference Schemes for Second-Order Equations 193
   8.3 Boundary Conditions for Second-Order Equations 199
   8.4 Second-Order Equations in Two and Three Dimensions 202

9 Analysis of Well-Posed and Stable Problems 205
   9.1 The Theory of Well-Posed Initial Value Problems 205
   9.2 Well-Posed Systems of Equations 213
   9.3 Estimates for Inhomogeneous Problems 223
   9.4 The Kreiss Matrix Theorem 225

10 Convergence Estimates for Initial Value Problems 235
   10.1 Convergence Estimates for Smooth Initial Functions 235
   10.2 Related Topics 248
   10.3 Convergence Estimates for Nonsmooth Initial Functions 252
   10.4 Convergence Estimates for Parabolic Differential Equations 259
   10.5 The Lax–Richtmyer Equivalence Theorem 262
   10.6 Analysis of Multistep Schemes 267
   10.7 Convergence Estimates for Second-Order Differential Equations 270
Contents

11 Well-Posed and Stable Initial-Boundary Value Problems 275
  11.1 Preliminaries 275
  11.2 Analysis of Boundary Conditions for the Leapfrog Scheme 281
  11.3 The General Analysis of Boundary Conditions 288
  11.4 Initial-Boundary Value Problems for Partial Differential Equations 300
  11.5 The Matrix Method for Analyzing Stability 307

12 Elliptic Partial Differential Equations and Difference Schemes 311
  12.1 Overview of Elliptic Partial Differential Equations 311
  12.2 Regularity Estimates for Elliptic Equations 315
  12.3 Maximum Principles 317
  12.4 Boundary Conditions for Elliptic Equations 322
  12.5 Finite Difference Schemes for Poisson's Equation 325
  12.6 Polar Coordinates 333
  12.7 Coordinate Changes and Finite Differences 335

13 Linear Iterative Methods 339
  13.1 Solving Finite Difference Schemes for Laplace's Equation in a Rectangle 339
  13.2 Eigenvalues of the Discrete Laplacian 342
  13.3 Analysis of the Jacobi and Gauss-Seidel Methods 345
  13.4 Convergence Analysis of Point SOR 351
  13.5 Consistently Ordered Matrices 357
  13.6 Linear Iterative Methods for Symmetric, Positive Definite Matrices 362
  13.7 The Neumann Boundary Value Problem 365

14 The Method of Steepest Descent and the Conjugate Gradient Method 373
  14.1 The Method of Steepest Descent 373
  14.2 The Conjugate Gradient Method 377
  14.3 Implementing the Conjugate Gradient Method 384
  14.4 A Convergence Estimate for the Conjugate Gradient Method 387
  14.5 The Preconditioned Conjugate Gradient Method 390

A Matrix and Vector Analysis 399
  A.1 Vector and Matrix Norms 399
  A.2 Analytic Functions of Matrices 406


---

B A Survey of Real Analysis 413
B.1 Topological Concepts 413
B.2 Measure Theory 413
B.3 Measurable Functions 414
B.4 Lebesgue Integration 415
B.5 Function Spaces 417

C A Survey of Results from Complex Analysis 419
C.1 Basic Definitions 419
C.2 Complex Integration 420
C.3 A Phragmen–Lindelöf Theorem 422
C.4 A Result for Parabolic Systems 424

References 427
Index 431