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# Differential Evolution

A Practical Approach to Global Optimization

With 292 Figures, 48 Tables and CD-ROM

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