### CONTENTS

1. **Introduction**
   1.1. Graphs and Graph Models .......................... 1
   1.2. Connected Graphs .................................. 9
   1.3. Common Classes of Graphs ......................... 19
   1.4. Multigraphs and Digraphs ......................... 26

2. **Degrees**
   2.1. The Degree of a Vertex ............................ 31
   2.2. Regular Graphs .................................... 38
   2.3. Degree Sequences .................................. 43
   2.4. Excursion: Graphs and Matrices .................. 48
   2.5. Exploration: Irregular Graphs ..................... 50

3. **Isomorphic Graphs**
   3.1. The Definition of Isomorphism ................. 55
   3.2. Isomorphism as a Relation ....................... 63
   3.3. Excursion: Graphs and Groups .................... 66
   3.4. Excursion: Reconstruction and Solvability ...... 76

4. **Trees**
   4.1. Bridges ........................................... 85
   4.2. Trees ............................................. 87
   4.3. The Minimum Spanning Tree Problem ............. 94
   4.4. Excursion: The Number of Spanning Trees ....... 101

5. **Connectivity**
   5.1. Cut-Vertices .................................... 107
   5.2. Blocks ........................................... 111
   5.3. Connectivity .................................... 115
   5.4. Menger’s Theorem ................................ 124
   5.5. Exploration: Geodetic Sets ..................... 130

6. **Traversability**
   6.1. Eulerian Graphs ................................ 133
   6.2. Hamiltonian Graphs .............................. 140
   6.3. Exploration: Hamiltonian Walks and Numbers .... 152
   6.4. Excursion: The Early Books of Graph Theory .... 156
7. Digraphs
   7.1. Strong Digraphs 161
   7.2. Tournaments 169
   7.3. Excursion: Decision-Making 176
   7.4. Exploration: Wine Bottle Problems 180

8. Matchings and Factorization
   8.1. Matchings 183
   8.2. Factorization 194
   8.3. Decompositions and Graceful Labelings 209
   8.4. Excursion: Instant Insanity 214
   8.5. Excursion: The Petersen Graph 219
   8.6. Exploration: γ-Labelings of Graphs 224

9. Planarity
   9.1. Planar Graphs 227
   9.2. Embedding Graphs on Surfaces 241
   9.3. Excursion: Graph Minors 249
   9.4. Exploration: Embedding Graphs in Graphs 253

10. Coloring
    10.1. The Four Color Problem 259
    10.2. Vertex Coloring 267
    10.3. Edge Coloring 280
    10.4. Excursion: The Heawood Map Coloring Theorem 288
    10.5. Exploration: Local Coloring 293

11. Ramsey Numbers
    11.1. The Ramsey Number of Graphs 297
    11.2. Turán’s Theorem 307
    11.3. Exploration: Rainbow Ramsey Numbers 314
    11.4. Excursion: Erdős Numbers 321

12. Distance
    12.1. The Center of a Graph 327
    12.2. Distant Vertices 333
    12.3. Excursion: Locating Numbers 341
    12.4. Excursion: Detour and Directed Distance 346
    12.5. Exploration: Channel Assignment 351
    12.6. Exploration: Distance Between Graphs 357
13. Domination

13.1. The Domination Number of a Graph 361
13.2. Exploration: Stratification 372
13.3. Exploration: Lights Out 377
13.4. Excursion: And Still It Grows More Colorful 381

Appendix 1. Sets and Logic 383
Appendix 2. Equivalence Relations and Functions 387
Appendix 3. Methods of Proof 391
Solutions and Hints for Odd-Numbered Exercises 397
References 425
Index of Names 437
Index of Mathematical Terms 440
List of Symbols 447