Contents

Preface .............................................. xvi

CHAPTER 1
Introduction ....................................... 1
1.1 Aquifers, Ground Water and Oil Reservoirs .......... 1
  1.1.1 Definitions .................................. 1
  1.1.2 The Moisture Distribution in a Vertical Profile .. 2
  1.1.3 Classification of Aquifers ..................... 4
  1.1.4 Properties of Aquifers ....................... 7
  1.1.5 The Oil Reservoir ............................ 8
1.2 The Porous Medium .............................. 13
1.3 The Continuum Approach to Porous Media .......... 15
  1.3.1 The Molecular and Microscopic Levels .......... 15
  1.3.2 Porosity and Representative Elementary Volume .. 19
  1.3.3 Areal and Linear Porosities .................. 21
  1.3.4 Velocity and Specific Discharge ............... 22
  1.3.5 Concluding Remarks ......................... 24

CHAPTER 2
Fluids and Porous Matrix Properties ................. 27
2.1 Fluid Density .................................... 27
  2.1.1 Definitions .................................. 27
  2.1.2 Mixture of Fluids ............................ 30
  2.1.3 Measurement of Density ....................... 31
2.2 Fluid Viscosity .................................. 32
  2.2.1 Definition .................................... 32
  2.2.2 Non-Newtonian Fluids ....................... 33
  2.2.3 Units ......................................... 34
  2.2.4 Effect of Pressure and Temperature .......... 34
  2.2.5 Measurement of Viscosity ..................... 35
2.3 Fluid Compressibility ........................... 37
2.4 Statistical Description of Porous Media .......... 38
  2.4.1 Particle-Size Distribution ................... 39
  2.4.2 Pore-Size Distribution ....................... 41
  2.4.3 Other Statistical Descriptions ............... 42
2.5 Porosity ................................................. 43
  2.5.1 Porosity and Effective Porosity ................ 43
  2.5.2 Porosity, Structure and Packing ............... 45
  2.5.3 Porosity Measurement ......................... 47
2.6 Specific Surface ........................................ 50
  2.6.1 Definitions ........................................ 50
  2.6.2 Measurement of Specific Surface .............. 51
2.7 Matrix and Medium Compressibility ................ 52

CHAPTER 3
Pressure and Piezometric Head ............................. 59
  3.1 Stress at a Point ...................................... 59
  3.2 Hydrostatic Pressure Distribution ................. 62
  3.3 Piezometric Head ..................................... 63

CHAPTER 4
The Fundamental Fluid Transport Equations in Porous Media ... 65
  4.1 Particles, Velocities and Fluxes in a Fluid Continuum ... 65
    4.1.1 Definitions of Particles and Velocities ....... 65
    4.1.2 Diffusive Velocities and Fluxes .............. 68
    4.1.3 The Eulerian and Lagrangian Points of View ... 70
    4.1.4 The Substantial Derivative ................... 71
  4.2 The General Conservation Principle .................. 74
  4.3 Equations of Mass, Momentum and Energy Conservation in a Fluid Continuum ..................... 77
    4.3.1 Mass Conservation of a Species ............... 77
    4.3.2 Mass Conservation of a Fluid System ......... 78
    4.3.3 Conservation of Linear Momentum of a Species \( \alpha \) .................. 79
    4.3.4 Conservation of Linear Momentum of a Fluid System .................. 80
  4.4 Constitutive Assumptions and Coupled Processes .......... 82
    4.4.1 General Considerations ....................... 82
    4.4.2 Principles to be Used in Forming Constitutive Equations ............ 84
    4.4.3 Coupled Processes .............................. 85
  4.5 A Porous Medium Model ................................ 90
    4.5.1 The Conceptual Model Approach ................ 90
    4.5.2 A Model of Flow Through a Porous Medium .... 92
    4.5.3 Frames of Reference ............................ 93
    4.5.4 An Averaging Procedure ....................... 95
  4.6 Equations of Volume and Mass Conservation ............ 98
    4.6.1 Equation of Volume Conservation ............... 98
    4.6.2 Equation of Mass Conservation of a Species in Solution .......... 100
    4.6.3 Equation of Mass Conservation ................. 102
  4.7 Equation of Motion .................................... 104
CHAPTER 5
The Equation of Motion of a Homogeneous Fluid

5.1 The Experimental Law of Darcy
5.2 Generalization of Darcy's Law
  5.2.1 Isotropic Medium
  5.2.2 Anisotropic Medium
5.3 Deviations from Darcy's Law
  5.3.1 The Upper Limit
  5.3.2 The Lower Limit
  5.3.3 The Slip Phenomenon
5.4 Rotational and Irrotational Motion
  5.4.1 The Potential and Pseudopotential
  5.4.2 Irrotational Flow
5.5 Hydraulic Conductivity of Isotropic Media
  5.5.1 Hydraulic Conductivity and Permeability
  5.5.2 Units and Examples
5.6 Anisotropic Permeability
  5.6.1 The Principal Directions
  5.6.2 Directional Permeability
5.7 Measurement of Hydraulic Conductivity
  5.7.1 General
  5.7.2 The Constant Head Permeameter
  5.7.3 The Falling Head Permeameter
  5.7.4 Determining Anisotropic Hydraulic Conductivity
5.8 Layered Porous Media
  5.8.1 Flow Normal and Parallel to the Medium Layers
  5.8.2 Equivalent Hydraulic Conductivity of Arbitrarily Directed Flow
  5.8.3 A Layered Medium as an Equivalent Anisotropic Medium
  5.8.4 Girinskii's Potential
5.9 Compressible Fluids
5.10 Derivations of Darcy's Law
  5.10.1 Capillary Tube Models
  5.10.2 Fissure Models
  5.10.3 Hydraulic Radius Models
  5.10.4 Resistance to Flow Models
  5.10.5 Statistical Models
  5.10.6 Averaging the Navier-Stokes Equations
  5.10.7 Ferrandon's Model

4.8 Tortuosity and Permeability
  4.8.1 Relationship Between Tortuosity and Permeability
  4.8.2 Tortuosity and Other Transport Coefficients
  4.8.3 Formation Factor and Resistivity Index (Amyx 1960) in Reservoir Engineering
7.1.1 Boundary of Prescribed Potential ........................................ 250
7.1.2 Boundary of Prescribed Flux ............................................ 251
7.1.3 The Steady Free (or Phreatic) Surface without Accretion ............ 252
7.1.4 The Unsteady Free (or Phreatic) Surface without Accretion .......... 254
7.1.5 The Steady Free (or Phreatic) Surface with Accretion ................ 256
7.1.6 The Unsteady Free (or Phreatic) Surface with Accretion .............. 258
7.1.7 Boundary of Saturated Zone (or of Capillary Fringe) .................. 259
7.1.8 The Seepage Face ..................................................... 260
7.1.9 Capillary Exposed Faces ............................................... 262
7.1.10 Discontinuity in Permeability ........................................ 263
7.1.11 A Note on Anisotropic Media ........................................ 269
7.1.12 Boundary Conditions in Terms of Pressure or Density .............. 270
7.2 A Well Posed Problem ................................................................ 270
7.3 Description of Boundaries in the Hodograph Plane ........................ 272
7.3.1 The Hodograph Plane ...................................................... 272
7.3.2 Boundaries in the Hodograph Plane ..................................... 274
7.3.3 Examples of Hodograph Representation of Boundaries ................ 280
7.3.4 Intersection of Boundaries of Different Types .......................... 284
7.4 The Relations between Solutions of Flow Problems in Isotropic and Anisotropic Media ................................................................. 289
7.4.1 The Flow Equations ....................................................... 290
7.4.2 Relationships among Parameters in the Two Systems .................. 291
7.4.3 Examples ................................................................. 296
7.5 Superposition and Duhamel's Principles ..................................... 297
7.5.1 Superposition ............................................................. 297
7.5.2 Unsteady Flow with Boundary Conditions Independent of Time .... 299
7.5.3 Unsteady Flow with Time-Dependent Boundary Conditions ......... 300
7.6 Direct Integration in One-Dimensional Problems .......................... 301
7.6.1 Solution of the One-Dimensional Continuity Equation ................ 301
7.6.2 Advance of a Wetting Front .............................................. 303
7.7 The Method of Images ....................................................... 304
7.7.1 Principles ................................................................. 304
7.7.2 Examples ................................................................. 306
7.8 Methods Based on the Theory of Functions .................................. 312
7.8.1 Complex Variables and Analytic Functions .............................. 313
7.8.2 The Complex Potential and the Complex Specific Discharge .......... 316
7.8.3 Sources and Sinks ....................................................... 316
7.8.4 Conformal Mapping ..................................................... 324
7.8.5 The Schwarz-Christoffel Transformation ................................ 333
7.8.6 Fictitious Flow in the $\omega$-Plane .................................... 337
7.9 Numerical Methods .......................................................... 338
7.9.1 Method of Finite Differences ........................................... 338
7.9.2 The Method of Finite Elements .......................................... 346
7.9.3 Relaxation Methods ...................................................... 348
7.9.4 Schmidt's Graphic Method ........................................... 350
7.10 Flow Nets by Graphic Methods ..................................... 351

CHAPTER 8

Unconfined Flow and the Dupuit Approximation ................................ 361

8.1 The Dupuit Approximation ........................................... 361
8.1.1 The Dupuit Assumptions ........................................... 361
8.1.2 Examples of Application to Hydraulic Steady Flows in Homogeneous Media ...................................................... 366
8.1.3 Unconfined Flow in an Aquifer with Horizontal Stratification ........ 369
8.1.4 Unconfined Flow in an Aquifer with Vertical Strata ............... 372
8.1.5 Unconfined Flow in a Two-Dimensional Inhomogeneous Medium ....... 373

8.2 Continuity Equations Based on the Dupuit Approximation ................. 374
8.2.1 The Continuity Equation ........................................... 374
8.2.2 Boundary and Initial Conditions ................................... 378
8.2.3 Some Solutions of Forchheimer's Equation ......................... 379
8.2.4 Some Solutions of Boussinesq's Equation .......................... 381

8.3 The Hodograph Method ................................................ 388
8.3.1 The Functions $\omega$ and $\bar{\omega}$ .................................. 388
8.3.2 The Hodograph Method ............................................. 389
8.3.3 Examples without a Seepage Face ................................... 391
8.3.4 Hamel's Mapping Function ......................................... 398
8.3.5 Zhukovski's and Other Mapping Functions ......................... 403
8.3.6 A Graphic Solution of the Hodograph Plane ....................... 406

8.4 Linearization Techniques and Solutions ............................... 408
8.4.1 First Method of Linearization of the Boussinesq Equation ........... 408
8.4.2 The Second Method of Linearization of the Boussinesq Equation .... 417
8.4.3 The Third Method of Linearization of the Boussinesq Equation ...... 419
8.4.4 The Method of Successive Steady States ........................... 420
8.4.5 The Method of Small Perturbations ................................ 422
8.4.6 The Shallow Flow Approximation .................................. 430

CHAPTER 9

Flow of Immiscible Fluids ................................................ 439

9.1 Introduction .......................................................... 439
9.1.1 Types of Two-Fluid Flows ....................................... 439
9.1.2 The Abrupt Interface Approximation .............................. 439
9.1.3 Occurrence ......................................................... 440

9.2 Interfacial Tension and Capillary Pressure ............................ 441
9.2.1 Saturation and Fluid Content ..................................... 441
9.2.2 Interfacial Tension and Wettability ................................ 441
9.2.3 Capillary Pressure ................................................ 444
9.2.4 Drainage and Imbibition ......................................... 449
9.2.5 Saturation Discontinuity at a Medium Discontinuity 452
9.2.6 Laboratory Measurement of Capillary Pressure 453
9.3 Simultaneous Flow of Two Immiscible Fluids 457
  9.3.1 The Basic Motion Equations 457
  9.3.2 Relative Permeability 459
  9.3.3 Mass Conservation in Multiphase Flow 466
  9.3.4 Statement of the Multiphase Flow Problem 466
  9.3.5 The Buckley-Leverett Equations 468
  9.3.6 Simultaneous Flow of a Liquid and a Gas 472
  9.3.7 Laboratory Determination of Relative Permeability 473
9.4 Unsaturated Flow 474
  9.4.1 Capillary Pressure and Retention Curve 475
  9.4.2 The Capillary Fringe 480
  9.4.3 Field Capacity and Specific Yield 483
  9.4.4 The Motion Equation 487
  9.4.5 Relative Permeability of Unsaturated Soils 491
  9.4.6 The Continuity Equation 495
  9.4.7 Methods of Solution and Examples 503
  9.4.8 Additional Comments on Infiltration and Redistribution of Moisture 513
  9.4.9 Comments on Vapor Movement in Unsaturated Flow 515
9.5 Immiscible Displacement with an Abrupt Interface 519
  9.5.1 The Abrupt Interface Approximation 519
  9.5.2 Piezometric Heads and Dynamic Equilibrium Conditions at a Stationary Interface 521
  9.5.3 The Boundary Conditions along an Interface 524
  9.5.4 Horizontal Interface Displacement 526
  9.5.5 Interface Displacement in the Vertical Plane 533
  9.5.6 Numerical and Graphic Methods 536
  9.5.7 Approximate Solutions based on Linearization 538
  9.5.8 Interface Stability 544
9.6 Determining the Steady Interface by the Hodograph Method 547
  9.6.1 Boundary Conditions 548
  9.6.2 Description of Boundaries in the Hodograph Plane 549
  9.6.3 Examples 549
9.7 The Interface in a Coastal Aquifer 557
  9.7.1 Occurrence 557
  9.7.2 The Ghyben-Herzberg Approximation 559
  9.7.3 Determining the Shape of a Stationary Interface by the Dupuit-Ghyben-Herzberg Approximation 561
  9.7.4 Approximate Solution for the Moving Interface 563
  9.7.5 Interface Upconing 569
  9.7.6 The Dupuit-Ghyben-Herzberg Approximation for an Unsteady Interface in a Thick Aquifer 573
CHAPTER 10

Hydrodynamic Dispersion ........................................... 579
10.1 Definition of Hydrodynamic Dispersion .................... 579
10.2 Occurrence of Dispersion Phenomena ..................... 582
10.3 Review of Some Hydrodynamic Dispersion Theories ....... 582
  10.3.1 Capillary Tube and Cell Models .................. 583
  10.3.2 Statistical Models ............................... 587
  10.3.3 Spatial Averaging ................................. 603
10.4 Parameters of Dispersion .................................... 605
  10.4.1 The Coefficients of Mechanical Dispersion and Hydrodynamic Dispersion .................. 605
  10.4.2 The Medium's Dispersivity ..................... 611
  10.4.3 Dispersivity-Permeability Relationship ............ 615
10.5 The Governing Equations and Boundary Conditions ....... 617
  10.5.1 The Partial Differential Equation in Cartesian Coordinates .................. 617
  10.5.2 The Partial Differential Equation in Curvilinear Coordinates ................. 619
  10.5.3 Initial and Boundary Conditions .................. 622
  10.5.4 Solving the Boundary Value Problems ............... 624
  10.5.5 The Use of Nondimensional Variables ............... 626
10.6 Some Solved Problems ....................................... 626
  10.6.1 One-dimensional Flow ............................. 627
  10.6.2 Uniform Flow in a Plane .......................... 633
  10.6.3 Plane Radial Flow ................................ 634
10.7 Heat and Mass Transfer .................................... 641
  10.7.1 Modes of Heat Transfer in a Porous Medium ........ 641
  10.7.2 Formulation of the Problem of Heat and Mass Transfer in a Fluid Continuum .... 643
  10.7.3 Formulation of the Problem of Heat and Mass Transfer in a Porous Medium .... 644
  10.7.4 Comments on Some Heat and Mass Transfer Coefficients .................. 647
  10.7.5 Simplifying the Macroscopic Heat and Mass Transfer Equations ............... 651
  10.7.6 Convective Currents and Instability ................ 653
  10.7.7 Some Similitude Considerations ................... 660

CHAPTER 11

Models and Analogs ............................................... 665
11.1 General .................................................. 665
11.2 Scaling Principles and Procedure ......................... 668
  11.2.1 The Two Systems .................................. 668
  11.2.2 Geometric Similarity ............................. 669
  11.2.3 Kinematic Similarity ............................. 670
  11.2.4 Dynamic Similarity .............................. 670
  11.2.5 Dimensional Analysis ............................ 671
11.2.6 Inspectional Analysis .............................................. 673
11.2.7 Modified Inspectional Analysis .................................. 676
11.3 The Sand Box Model .................................................. 678
  11.3.1 Description ..................................................... 678
  11.3.2 Scales .......................................................... 680
11.4 The Viscous Flow Analogs .......................................... 687
  11.4.1 General ........................................................ 687
  11.4.2 Description of the Vertical Hele-Shaw Analog .............. 687
  11.4.3 Establishing the Analogy between Analog and Prototype .... 690
  11.4.4 Scales for the Vertical Analog ................................ 693
  11.4.5 Recommended Applications of Vertical Analog .............. 696
  11.4.6 The Liquids .................................................... 697
  11.4.7 The Horizontal Hele-Shaw Analog—Description and Scales ... 697
  11.4.8 Simulation of an Infinite Horizontal Aquifer ............... 701
11.5 Electric Analogs .................................................... 702
  11.5.1 Description of the Electrolytic Tank and the Conducting Paper Analogs ............................................. 702
  11.5.2 Scales for the Electrolytic Tank Analog ..................... 708
  11.5.3 The Resistance Network Analog for Steady Flow .......... 710
  11.5.4 The Resistance-Capacitance Network for Unsteady Flow ... 716
  11.5.5 The Ion Motion Analog ........................................ 719
11.6 The Membrane Analog ............................................... 722
11.7 Summary .................................................................. 725
Answers to Exercises ...................................................... 729
Bibliography .................................................................. 733
Index ........................................................................... 757