Index

2R planar manipulator
control, 652
DH transformation matrix, 212
dynamics, 491, 540
equations of motion, 494
forward acceleration, 439
ideal, 491
inverse acceleration, 441
inverse kinematics, 281, 286, 399
inverse velocity, 366, 368
Jacobian matrix, 350, 352
joint 2 acceleration, 433
joint path, 591
kinetic energy, 492
Lagrange dynamics, 533, 542
Lagrangean, 493
Newton-Euler dynamics, 516
potential energy, 493
recursive dynamics, 524
time-optimal control, 630
with massive links, 542

3R planar manipulator
DH transformation matrix, 204
forward kinematics, 227

4R planar manipulator
statics, 548

Acceleration
angular, 423, 428, 429, 431, 432
bias vector, 440
body point, 317, 432, 434, 452
centripetal, 432
constant parabola, 593
constant path, 580
Coriolis, 454
discontinuous path, 588
discrete equation, 620, 631
end-effector, 429
forward kinematics, 437, 439
gravitational, 532, 538, 549
inverse kinematics, 439
jump, 573
matrix, 414, 423, 434–436
recursive, 507, 510
sensors, 659
tangential, 432
Active transformation, 72
Actuator, 7, 12
force and torque, 513, 529, 553
optimal torque, 632, 633
torque equation, 518, 630
Algorithm
floating-time, 619, 629
inverse kinematics, 286
LU factorization, 380
LU solution, 380
Newton-Raphson, 398
Angle-axis rotation, 106
Angular acceleration, 423, 431, 432
combination, 428
end-effector, 429
in terms of Euler parameters, 429, 431
in terms of quaternion, 431
recursive, 414
Angular momentum
2 link manipulator, 462
Angular velocity, 53, 56, 57, 86, 299, 306
alternative definition, 318
combination, 305
coordinate transformation, 308
decomposition, 305
elements of matrix, 311
in terms of Euler parameters, 310
in terms of quaternion, 309
in terms of rotation matrix, 307
instantaneous, 301
instantaneous axis, 302
matrix, 300
principal matrix, 304
recursive, 412, 509
Articulated arm, 8, 231, 267, 357, 408
Atan2 function, 272
Automorphism, 102
Axis-angle rotation, 81, 84, 85, 90, 91, 94

Block diagram, 644
Brachistochrone, 616, 627
Bryant angles, 58

Cardan
angles, 58
frequencies, 58
Cartesian
angular velocity, 56
defect-effector position, 365
defect-effector velocity, 366
manipulator, 8, 11
path, 592
Central difference, 625
Chasles theorem, 154, 166
Christoffel operator, 488, 535
Co-state variable, 610
Control
adaptive, 649
admissible, 618
bang-bang, 609, 610
characteristic equation, 646
closed-loop, 643
command, 643
computed force, 651
computed torque, 648, 649
derivative, 655
desired path, 643
error, 643
feedback, 644
feedback command, 651
feedback linearization, 648, 651
feedforward command, 651
gain-scheduling, 649
input, 650
integral, 655
linear, 649, 654
minimum time, 609
modified PD, 657
open-loop, 643, 650
path points, 595
PD, 657
proportional, 654
robots, 13
sensing, 657
stability of linear, 646
time-optimal, 618, 622, 629, 630, 633
time-optimal description, 618
time-optimal path, 627
Controller, 7
Coordinate
cylindrical, 152
frame, 17
non-Cartesian, 487
non-orthogonal, 117
parabolic, 487
spherical, 153, 332
system, 17
Coriolis
acceleration, 428, 434
effect, 454
force, 453
Cycloid, 617

Denavit-Hartenberg, 31
method, 199, 202, 248
nonstandard method, 223, 283
notation, 199
parameters, 199, 334, 345, 510, 548
transformation, 208, 212–218, 220, 222, 243
Differential manifold, 71
Differentiating, 312
   B-derivative, 312, 314
   G-derivative, 312, 317
second, 320
   transformation formula, 317
Distal end, 199, 548
Dynamics, 421, 507
   2R planar manipulator, 516, 524
   4 bar linkage, 514
   actuator’s force and torque, 529
   backward Newton-Euler, 522
   forward Newton-Euler, 529
   global Newton-Euler, 511
Newton-Euler, 511
   one-link manipulator, 513
   recursive Newton-Euler, 511, 522

Earth
   effect of rotation, 453
   kinetic energy, 486
   revolution, 486
   rotation, 486
   rotation effect, 428
Eigenvalue, 87
Eigenvector, 87
Ellipsoid
   energy, 465
   momentum, 464
End-effector, 6
   acceleration, 437
   angular acceleration, 429
   angular velocity, 363
   articulated robot, 267
   configuration vector, 348, 405, 437
   configuration velocity, 437
   force, 530
   frame, 207, 231
inverse kinematics, 265
kinematics, 237
link, 199
orientation, 271, 364
path, 591, 600
position, 231
position kinematics, 226
position vector, 358
rotation, 597
SCARA position, 149
SCARA robot, 240
space station manipulator, 243
spherical robot, 247
time optimal control, 609
velocity, 348, 354, 365
velocity vector, 348

Energy
   Earth kinetic, 486
   kinetic rigid body, 461
   kinetic rotational, 458
   link’s kinetic, 531, 537
   link’s potential, 532
   mechanical, 486
   point kinetic, 451
   potential, 489
   robot kinetic, 531, 538
   robot potential, 532, 538
Euler
   -Lexell-Rodriguez formula, 83
   angles, 18, 48, 51, 53, 107
   integrability, 57
   coordinate frame, 56
   equation of motion, 457, 460, 461, 466, 467, 513, 523
   frequencies, 53, 56, 306
   inverse matrix, 69
   parameters, 88–92, 96–98, 100, 111, 309, 310
   rotation matrix, 51, 69
   theorem, 48, 88
Euler equation
   body frame, 460, 467
Euler-Lagrange
   equation of motion, 614, 615
Eulerian
viewpoint, 326

Floating time, 620
  1 DOF algorithm, 619
  analytic calculation, 627
  backward path, 622
  convergence, 625
  forward path, 621
  method, 618
  multi DOF algorithm, 629
  multiple switching, 633
  path planning, 627
  robot control, 629

Force, 449
  action, 512
  actuator, 529
  conservative, 489
  Coriolis, 454
  driven, 512
  driving, 512
  generalized, 483, 532
  gravitational vector, 533
  potential, 489
  potential field, 485
  reaction, 512
  sensors, 660
  shaking, 516
  time varying, 454

Forward kinematics, 32

Frame
  central, 455
  final, 207
  goal, 207
  principal, 457
  reference, 16
  special, 206
  station, 206
  tool, 207
  world, 206
  wrist, 207

Generalized
  coordinate, 480, 483, 484, 490
  force, 482, 483, 485, 487, 489, 491, 494, 530

  inverse Jacobian, 403
  Grassmanian, 177
  Group properties, 70
  Hamiltonian, 610
  Hand, 231
  Hayati-Roberts notation, 224
  Helix, 154
  Homogeneous
    compound transformation, 145
    coordinate, 133, 138
    direction, 138
    general transformation, 139, 143
    inverse transformation, 139, 141, 142, 146
    position vector, 133
    scale factor, 133
    transformation, 131, 134–137, 139, 141

  Integrability, 57
  Inverse kinematics, 32, 265
    decoupling technique, 265
    inverse transformation technique, 272
    iterative technique, 284
    Pieper technique, 274
  Inverted pendulum, 652

  Jacobian
    analytical, 365
    elements, 363
    generating vector, 353, 355, 404
    geometrical, 365
    inverse, 287, 403
    of link, 531
    polar manipulator, 349

  Jerk
angular, 430  
matrix, 436  
transformation, 435, 437  
zero path, 579  

Joint, 3  
acceleration vector, 437  
active, 4  
coordinate, 4  
cylindrical, 252  
inactive, 4  
orthogonal, 8  
parallel, 8  
passive, 4  
path, 591  
perpendicular, 8  
screw, 4  
variable vector, 348  
velocity vector, 348, 355  

Joint angle, 200  
Joint distance, 200  
Joint parameters, 202  

Kinematic length, 200  
Kinematics, 31  
acceleration, 423  
forward, 32, 226  
forward acceleration, 437  
forward velocity, 348  
inverse, 32, 265, 272  
inverse acceleration, 439  
inverse velocity, 365  
numerical methods, 377  
velocity, 345  

Kinetic energy, 451  
Earth, 486  
link, 537  
parabolic coordinate, 487  
rigid body, 461  
robot, 531, 538  
rotational body, 458  

Kronecker’s delta, 65, 457, 479  

Lagrange  
dynamics, 530  
equation, 536  
equation of motion, 480, 489  
mechanics, 489  
multiplier, 617  

Lagrange equation  
explicit form, 488  

Lagrangean, 489, 538  
robot, 538  
viewpoint, 326  

Law  
motion, 450  
motion second, 450, 455  
motion third, 450  
robotics, 1  

Levi-Civita density, 96  
Lie group, 71  

Link, 3  
angular velocity, 346  
class 1 and 2, 213  
class 11 and 12, 218  
class 3 and 4, 214  
class 5 and 6, 215  
class 7 and 8, 216  
class 9 and 10, 217  
classification, 219  
end-effector, 199  
Euler equation, 523  
kinetic energy, 531  
Newton-Euler dynamics, 511  
recursive acceleration, 507, 510  
recursive Newton-Euler dynamics, 522  
recursive velocity, 509, 510  
rotational acceleration, 508  
translational acceleration, 508  
translational velocity, 347  
velocity, 345  

Link length, 200  
Link offset, 200  
Link parameters, 202  
Link twist, 200  
Location vector, 156, 158  

LU factorization method, 377, 392  

Manipulator  
2R planar, 491, 533
3R planar, 227
articulated, 205
definition, 5
inertia matrix, 532
one-link, 490
one-link control, 655
one-link dynamics, 513
PUMA, 204
SCARA, 8
transformation matrix, 267
Mass center, 450, 451, 455
Matrix
  skew symmetric, 68, 69, 82, 89
Moment, 449
  action, 512
  driven, 512
  driving, 512
  reaction, 512
Moment of inertia
  about a line, 479
  about a plane, 479
  about a point, 479
  characteristic equation, 477
  diagonal elements, 477
  Huygens-Steiner theorem, 471
  matrix, 468
  parallel-axes theorem, 469
  polar, 468
  principal, 469
  principal axes, 458
  principal invariants, 477
  product, 468
  pseudo matrix, 469
  rigid body, 457
  rotated-axes theorem, 469
Moment of momentum, 450
Momentum, 450
  angular, 450
  ellipsoid, 464
  linear, 450
Motion, 14

Newton
  equation of motion, 480

Newton equation
  body frame, 456
  global frame, 455
  Lagrange form, 482
  rotating frame, 453
Newton-Euler
  backward equations, 522
  equation of motion, 523
  forward equations, 529
  global equations, 511
  recursive equations, 522
Numerical methods, 377
  analytic inversion, 394
  Cayley-Hamilton inversion, 395
  condition number, 388
  ill-conditioned, 388
  Jacobian matrix, 404
  LU factorization, 377
  LU factorization with pivoting, 383
  matrix inversion, 390
  Newton-Raphson, 398, 400
  nonlinear equations, 397
  norm of a matrix, 389
  partitioning inversion, 393
  uniqueness of solution, 387
  well-conditioned, 388
Optimal control, 609
  a linear system, 610
  description, 618
  first variation, 615
  Hamiltonian, 610, 613
  Lagrange equation, 614
  objective function, 609, 613
  performance index, 613
  second variation, 615
  switching point, 611
Orthogonality condition, 64
Passive transformation, 72
Path
  Brachistochrone, 627
  Cartesian, 592
  constant acceleration, 580
constant angular acceleration, 599
control points, 595
cubic, 571
cycloid, 590
harmonic, 589
higher polynomial, 578
jerk zero, 579
joint space, 591
non-polynomial, 589
planning, 592
point sequence, 582
quadratic, 577
quintic, 578
rest-to-rest, 573, 574
rotational, 597
splitting, 584
to-rest, 573

Pendulum
control, 652
inverted, 652, 657
linear control, 655
oscillating, 484
simple, 425, 483
spherical, 490

Permutation symbol, 96
Phase plane, 611
Pieper technique, 274

Plücker
angle, 181
classification coordinate, 178
distance, 181
line coordinate, 173, 175–177, 181, 185–187, 247, 248
moment, 180
ray coordinate, 175, 177
reciprocal product, 181
screw, 186
virtual product, 181

Poinstot's construction, 464
Point at infinity, 138
Pole, 163
Position sensors, 658
Positioning, 14

Potential
force, 489
Potential energy
robot, 532, 538
Proximal end, 199, 548

Quaternions, 99
addition, 99
composition rotation, 102
flag form, 99
inverse rotation, 101
multiplication, 99
rotation, 100

Rigid body
acceleration, 431, 508
angular momentum, 458
angular velocity, 86
Euler equation of motion, 461, 466
kinematics, 127
kinetic energy, 461
moment of inertia, 457
motion, 127
motion classification, 167
motion composition, 131
principal rotation matrix, 476
rotational kinetics, 457
steady rotation, 462
translational kinetics, 455
velocity, 321, 323

Robot
application, 13
articulated, 8, 231, 238, 267, 357, 361
Cartesian, 11
classification, 7
control, 13, 14
control algorithms, 648
cylindrical, 11, 259
dynamics, 14, 19, 507, 533
dynamic equilibrium, 600
equation of motion, 540
forward kinematics, 226, 246
gravitational vector, 533
inertia matrix, 532
kinematics, 14
kinetic energy, 531, 538
Lagrange dynamics, 530, 536
Lagrange equation, 533
Lagrangian, 532, 536
link classification, 245
modified PD control, 657
Newton-Euler dynamics, 511
PD control, 657
potential energy, 532, 538
recursive Newton-Euler dynamics, 522
rest position, 200, 203, 231, 235, 239
SCARA, 149, 239
spherical, 10, 205, 235, 246, 274, 355
state equation, 613
statics, 546
time-optimal control, 613, 629
velocity coupling vector, 533
Robotic
gamey, 8
history, 1
law, 1
Rodriguez
rotation formula, 83, 84, 89, 92–95, 101, 106, 114, 128, 158, 161, 167, 172, 302, 337, 597
vector, 95, 113
Roll-pitch-yaw
frequency, 60
global angles, 41, 59
global rotation matrix, 41, 59
Rotation, 32, 83
about global axis, 33, 38, 40
about local axis, 43, 47, 48
angle-axis, 106
axis-angle, 81, 83–85, 90, 91, 94, 106
composition, 113
decomposition, 113
eigenvalue, 87
eigenvector, 87
exponential form, 93
general, 63
infinitesimal, 92
local versus global, 61
matrix, 18, 105
pole, 326
quaternion, 100
stanley method, 98
X-matrix, 33
x-matrix, 43
Y-matrix, 33
y-matrix, 43
Z-matrix, 33
z-matrix, 43
Rotational path, 597
Rotator, 83, 102
SCARA
manipulator, 8
robot, 149, 239
Screw, 154, 157, 166
axis, 154
central, 155, 156, 159, 160, 173, 187, 202, 243, 245, 247
combination, 170, 172
coordinate, 154
decomposition, 172, 173
exponential, 171
forward kinematics, 243
instantaneous, 187
intersection, 248
inverse, 169, 170, 172
left-handed, 155
link classification, 245
location vector, 156
motion, 202, 327
parameters, 155, 164
pitch, 154
Plücker coordinate, 186
principal, 166, 172, 173
reverse central, 156
right-handed, 15, 155
special case, 162
transformation, 158, 165
twist, 154
Second derivative, 320
Sensor
  acceleration, 659
  position, 658
  rotary, 658
  velocity, 659
Sheth notation, 248
Singular configuration, 291
Spherical coordinate, 153
Spinor, 83, 102
Spline, 588
Stanley method, 98
Stark effect, 487
Symbols, xi

Tilt vector, 231
Time derivative, 312
Top, 53
Transformation, 31
  active and passive, 71
  general, 63
  homogeneous, 131
Transformation matrix
  derivative, 332
  differential, 336, 337
  elements, 66
  velocity, 327
Translation, 32
Triad, 15
Trigonometric equation, 271
Turn vector, 231
Twist vector, 231

Unit system, xi
Unit vectors, 16

Vector
  gravitational force, 533
  velocity coupling, 533
vector
  gravitational force, 537
  velocity coupling, 536
Vector decomposition, 117
Velocity
  body point, 452
  discrete equation, 620, 631
  end-effector, 348
  inverse transformation, 330
  matrix, 436
  operator matrix, 333
  prismatic transformation, 335
  revolute transformation, 335
  sensors, 659
  transformation matrix, 327, 329, 331, 333
Work, 451, 454
  virtual, 483
Work-energy principle, 451
Workspace, 11
Wrench, 452
Wrist, 12-14, 231
  decoupling kinematics, 266
  forward kinematics, 229
  frame, 207
  kinematics assembly, 238
  point, 6, 229, 271
  position vector, 270
  spherical, 6, 205, 231, 235, 361
  transformation matrix, 230, 267

Zero velocity point, 326